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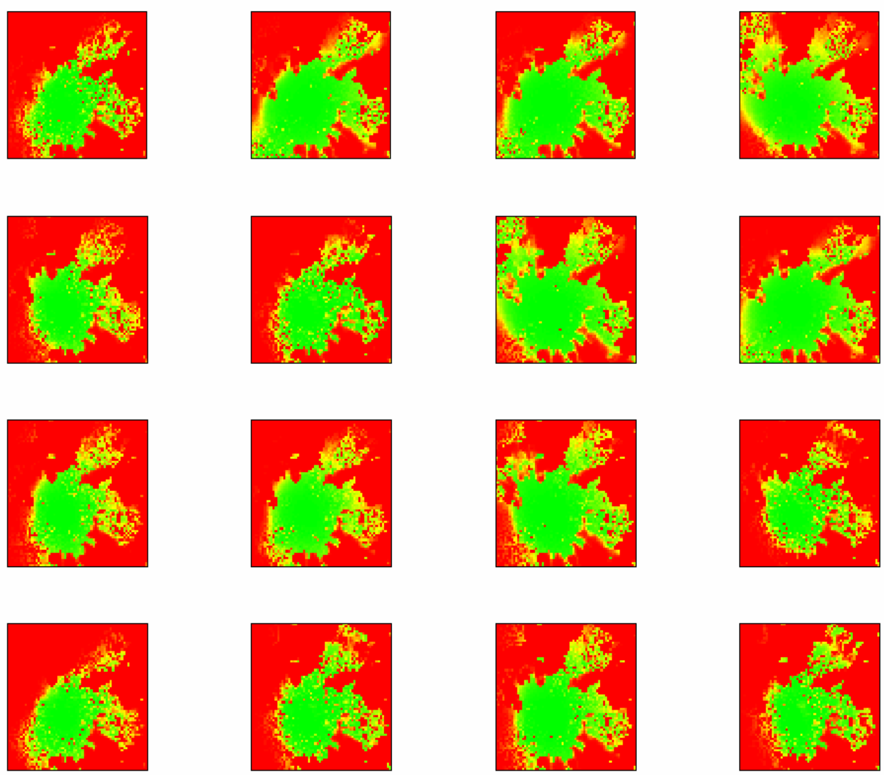
Addressing Uncertainty in Signal Propagation and Sensor Performance Predictions

D. Keith Wilson, Chris L. Pettit, Sean Mackay, Matthew S. Lewis, and Peter M. Seman

November 2008



COVER: Terrain and weather effects on the probability of detection for an aerial source. Shown are probability of detection calculations for 16 equally likely, randomized combinations of source height, noise background, ground permeability, wind speed, and wind direction.

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Abstract: As advanced sensors are increasingly relied upon for force protection, rapid strike, maneuver support, and other tasks, expert decision support tools (DSTs) are needed to recommend appropriate mixes of sensors and placements that will maximize their effectiveness. These tools should predict effects on sensor performance of the many complexities of the environment, such as terrain conditions, the atmospheric state, and background noise and clutter. However, the information available for such inputs is often incomplete and imprecise. To avoid drawing unwarranted conclusions from DSTs, the calculations should reflect a realistic degree of uncertainty in the inputs. In this report, a Bayesian probabilistic framework is developed for this purpose. The initial step involves incorporating uncertainty in the weather forecast, terrain state, and tactical situation by constructing an ensemble of scenarios. Next, a likelihood function for the signal propagation model parameters specifies uncertainty at smaller spatial scales, such as that caused by wind gusts, turbulence, clouds, vegetation, and buildings. An object-oriented software implementation of the framework is sketched. Examples illustrate the importance of uncertainty for optimal sensor selection and determining sensor coverage.

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Preface

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M. S. Lewis is an Oak Ridge Institute for Science and Education (ORISE) fellow and works with ERDC/Cold Regions Research and Engineering Laboratory (CRREL) in Hanover, NH under an ORISE contract. ORISE is managed by the Oak Ridge Associated Universities (U.S. Department of Energy).

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Unit Conversion Factors

Multiply	By	To Obtain
degrees Fahrenheit	(F-32)/1.8	degrees Celsius
feet	0.3048	meters
inches	0.0254	meters
pounds (force) per square foot	47.88026	pascals

1 Introduction: Why Be Concerned about Uncertainty?

As the U.S. Army relies increasingly upon advanced sensors for force protection, surveillance, rapid strike, maneuver support, and other tasks, expert decision support tools (DSTs) are needed to recommend appropriate mixes of sensors and placements that will maximize their effectiveness. These tools should predict the effects on sensor performance of the many complexities of the environment, such as terrain conditions, the atmospheric state, and background noise and clutter. Some previous efforts to develop advanced DSTs for terrain-based analyses of seismic/acoustic and infrared (IR) sensor performance are described by Wilson and Szeto (2000), Wilson (2006), Hieb et al. (2007), and Wilson et al. (2007).

The *reliability* of the recommendations from a DST is a key to well-informed decision making. Mission success and lives may hinge on the question “Can I trust these predictions?” The answer depends not just on whether the software is functioning properly and has been tested in limited, well-controlled circumstances; it also depends on whether the information being supplied to the software in particular, often unforeseen, circumstances is sufficient for reliable recommendations. In practice, the quality and completeness of available environmental and tactical information may be more important than the fidelity of the actual physical models upon which a DST is built. The traditional verification, validation, and accreditation (VV&A) process only partially addresses this concern. Selection of an appropriate sensing strategy in harsh, complex, and urban environments often requires system redundancy that is apparent only when imperfect inputs and modeling capabilities in DSTs are recognized.

A familiar illustration of predictive uncertainty is weather forecasting. Most everyone has been frustrated at one time or another by a forecast that proved incorrect. While recognizing the occasional shortcomings of weather forecasts, however, we continue to place great value in them. Over time, we develop a fairly good understanding of the forecast reliability. Sensor performance predictions may be viewed similarly. Such predictions depend not only on the weather, but on many other uncertain environmental and tactical factors, such as the local terrain state and background noise activity at a particular site. Instead of dismissing predictions based

on imperfect and incomplete knowledge, we should systematically assess the skill, or uncertainty, associated with the predictions.

The essence of the problem is that most current DSTs for sensor performance and signal propagation prediction treat the model inputs as exact, or “crisp,” information. If the wind speed at a particular location is forecast to be 6.789 m/s at 11:44 AM on next Tuesday, no probabilistic distribution, error bars, or “fuzziness” is associated with that information. The DST subsequently generates a single prediction based on the crisp input information, without providing any quantitative or qualitative information on the quality of the prediction.

The purpose of this report is to lay important groundwork for addressing uncertainty in predictions of signature propagation and sensor performance. Section 2 describes some practical examples where uncertainty plays an important role in signature propagation and sensor performance prediction. These examples are presented first to help motivate and constrain the discussion in the remainder of the report. Section 3 discusses the larger context of uncertainty classification and analysis, and how some of the examples mentioned in Section 2 fit into this broader context. Lastly, in Section 4, a probabilistic framework for dealing with uncertainty in signature propagation and sensor performance is suggested, along with a software architecture through which it might be implemented.

2 Practical Uncertainty Issues

Issues involving uncertainty in signature propagation and sensor performance predictions are discussed in this section. The list draws upon the authors' practical experience; it is not claimed to be complete and some of the issues overlap with each other. The main purpose is not to create a formal taxonomy, but rather to elucidate particular issues that should be considered. The list is:

1. Operational environment of the sensor differs from coarse-scale weather/terrain representation;
2. Wave propagation sensitivity to irresolvable and highly variable environmental details;
3. Varying resolutions in environmental datasets and gridding methods;
4. Inaccurate and missing environmental data;
5. Complex interrelationships among systems of environmental variables;
6. Random (chaotic) behavior of atmospheric and dependent processes;
7. Imperfect physics in wave propagation and signature models;
8. Correlated observations from suites of sensors;
9. Lack of knowledge of sensor properties and algorithms;
10. Complex, dynamic, unknown scenarios in which the sensors are used;
11. Uncertainty of when and where predictions are required.

In the following, we explain these issues in more detail and illustrate them with examples. Possible solutions are mentioned for some of the issues; the reader may notice that similar solutions often apply to multiple issues. Whichever solutions are considered, a sensitivity analysis ought to be conducted to assess and document the relative importance of feeding inaccurate or imprecise information to the associated computational models.

Issue 1: Operational environment of the sensor differs from coarse-scale weather/terrain representation

Explanation

Weather and terrain representations often assume that the environment is homogeneous at a very coarse scale. In actuality, many distinct environments may be present at scales below the resolution of the environmental data. This may be an issue in both the spatial and temporal sense.

Example

Weather forecasts from the U.S. Air Force Weather Agency now have a horizontal resolution of 15 km. A particular "grid cell" at this resolution may contain open plains, hilly areas, forests, cities, villages, parks, etc.

Possible solution

Attempt to supplement the coarse-scale environmental information with local information available to the analyst. For example, an analyst may know that sensor performance predictions for an urban area within the larger grid cell are desired. This provides an opportunity to tailor the predictions to the urban conditions.

Issue 2: Wave propagation sensitivity to irresolvable and highly variable environmental details

Explanation

Propagation of acoustic, seismic, and electromagnetic (EM) waves is sensitive, in principle, to environmental objects as small as about one-tenth of the wavelength. Such objects can include small turbulent eddies, vegetation, dust particles, and precipitation. Even with highly detailed atmospheric and terrain observations, it is generally not possible to characterize such small objects at a particular location and time. Therefore, the effect of these small variations in the environment cannot be calculated in a direct, deterministic sense.

Examples

- Acoustic scattering by small turbulent eddies raises sound levels in shadow regions and reduces signal coherence across microphone arrays (see Figure 1).
- Optical signals are scattered and attenuated by dust and other small particles in the atmosphere.
- Radio frequency (RF) signals undergo random fading as a result of scattering from terrain objects (such as buildings, vegetation, and random surface variations) and atmospheric turbulence.

Possible solutions

- Develop statistical models for the wave propagation that depend on average, observable quantities, such as dust concentration or turbulent kinetic energy dissipation rate. Similarly, develop more efficient models that propagate the *statistical moments* of the acoustic and RF signals, rather than deterministic realizations.
- Average wave propagation predictions from a large number of *random realizations* of the propagation medium. This is usually very computationally intensive. For example, one might estimate scattering into an acoustic shadow zone by synthesizing a large number of artificial turbulence fields.

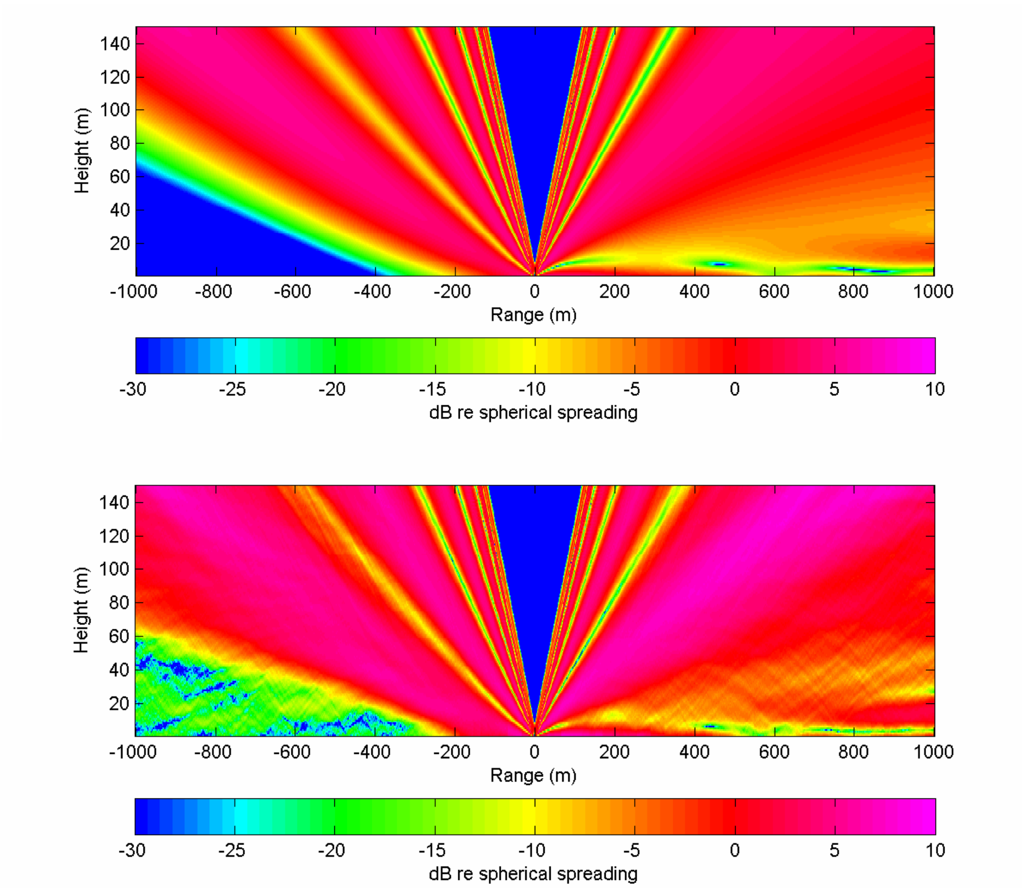

Figure 1. Propagation of sound upwind (to the left) and downwind (to the right) from a 100-Hz source at 5-m height. The top image shows a calculation without turbulent scattering; the bottom image includes a realization of atmospheric turbulence. It is seen that the sound field near the ground is enhanced and randomized by the turbulence, particularly in the upwind direction.

Issue 3: Varying resolutions and representations in environmental datasets

Explanation

The resolution of available environmental information varies widely. Weather predictions are available at a horizontal resolution of 1 km at best; 15 km is more typical. Terrain elevation data, on the other hand, are often available at a horizontal resolution of 3 to 30 m. Other datasets, such as those containing soil properties and cultural features, typically have resolutions of 100 to 500 m. The datasets may also rely on different representation techniques, such as regularly spaced points (a raster) or polygons.

Examples

In the U.S. Army Engineer Research and Development Center (ERDC) Battlefield Terrain Reasoning and Analysis software (Hieb et al. 2007), the meteorological forecast has very coarse resolution compared to terrain "complex," which is a polygonal, variable resolution representation of the terrain facets including ground properties. This creates dramatic "step" changes in predictions across forecast grid-cell boundaries that do not exist in actuality.

Possible solutions

Interpolation of coarse fields to make them smooth at finer scales of interest. Alternatively, one might interpolate the end product, namely the signature propagation or sensor performance calculations. In many cases, software tools are available to convert between different data gridding techniques. However, these are mostly cosmetic solutions: the coarser datasets will usually be unrealistically smooth when interpolated to a higher resolution.

Issue 4: Inaccurate and missing environmental data

Explanation

Environmental observations, whether of the atmosphere or terrain, are rarely fully accurate and complete. This is particularly so in many areas where military conflicts may occur.

Examples

- Very little information is available on subsurface characteristics of the earth affecting seismic wave propagation. This makes accurate prediction of seismic detection ranges difficult.
- The acoustic background noise spectrum plays a key role in acoustic detection (just as important as the actual signal level), but is often not known accurately. Factors such as time of day, proximity to roadways, and intermittent activity, can all be dominant.
- Fog, snow, and rain, which greatly affect the performance of optical and infrared sensors, often occur in very localized and transient patches. Therefore, it is often uncertain in advance whether the sensors will be operating in such impaired conditions or not.

Possible solutions

Attempt to assess the predictive skill of lower fidelity models through comparisons to higher fidelity ones or empirical data. If this is infeasible, the experience of experts may provide a rough assessment of predictive skill.

Issue 5: Complex interrelationships among systems of environmental variables

Explanation

It is generally difficult to describe uncertainties in the behavior of systems of variables with complex, nonlinear, interrelationships. The atmospheric environment is such a system.

Examples

- Variations in the earth's surface albedo (diffuse solar reflectivity) cause variations in ground heating, which are important to evaluating temperature contrasts for infrared sensing. These variations in ground heating help drive wind and turbulence. The turbulence transports heat, which again impacts temperatures of surfaces. Even if we could accurately describe the uncertainty in the surface albedo, modeling its subsequent effects and feedbacks on the other variables is extremely challenging.
- Ordinarily, the predictive skill of atmospheric models is assessed by calculating the standard deviation between the forecast and the ob-

served wind at a selected set of points. But sound propagation, among other processes, depends on multipoint correlations in the spatial structure of the wind field. These multipoint correlations are determined by complex, nonlinear processes. The standard deviation at a particular point in the flow does not provide the information necessary for a meaningful skill assessment.

Possible solution

Use realistic physical models, such as numerical weather forecasts and acoustic/EM wave propagation models, to capture relationships between the variables of interest. Introduce randomness in the output through Monte Carlo or Latin hypercube sampling (LHS) of combinations of multiple variables.

Issue 6: Random (chaotic) behavior of atmospheric and dependent processes

Explanation

The future state of the atmosphere cannot be predicted with certainty. As is now clearly recognized through chaos theory, small perturbations to the initial conditions, either through physical variations or through how these variations are represented in a computer model, can have a large impact on the true and forecasted future states.

Examples

When an acoustic propagation prediction is run for a single, crisp atmospheric forecast, the predictions may show very sharply delineated regions of low or high signal level. (An example is shown in Section 5, Figure 8.) The locations of these regions are very sensitive to details of the atmospheric data supplied to the model. When small perturbations are applied to the model inputs, the predictions are substantially smoothed.

Possible solution

Use ensemble forecast methods to predict a range of plausible environmental states (e.g., Kalnay 2003, Palmer 2006) and calculate propagation through these multiple states. In some cases, the distribution of forecasts and sensor performance predictions can be represented in an intuitive manner, such as the "fans" familiar in forecasts of hurricane tracks.

Issue 7: Imperfect physics in wave propagation and signature models

Explanation

Most practical models for wave propagation and signatures do not incorporate all of the physics that may be significant. Compromises in fidelity must be made to keep the demands on computational resources reasonable.

Examples

- Many RF propagation models do not include diffraction over terrain features, such as buildings or hills, or refraction by the atmosphere.
- Acoustical propagation models often do not incorporate reflections and diffractions around buildings in urban environments.
- Wave propagation models may not properly handle terrain heterogeneity, i.e., propagation paths that traverse multiple terrain types such as land and water.

Possible solutions

- Create more accurate, yet computationally efficient models. (This has usually been the subject of much previous research and is often very challenging.)
- Take advantage of increasing computational capabilities by employing more sophisticated propagation and signature models.
- Characterize the predictive skill associated with the computationally feasible, but less accurate, models by comparing them to experimental data or to computationally intensive, but more accurate, models. Develop methods to express the predictive skill of the less accurate models to their users.

Issue 8: Correlated observations from suites of sensors

Explanation

Most formulations for statistical hypothesis testing (as in signal detection problems) and sensor data fusion assume independent observations. This assumption greatly simplifies the mathematics. In real-world situations, however, there are often significant dependencies between sensor observations.

Examples

- Fog and high humidity hinder transmission of infrared and high-frequency acoustical signals.
- Two similar sensors that are placed close to one another may provide very similar, highly correlated information.

Possible solution

Recent research is now beginning to address this problem, e.g., Gananapandithan and Natarajan (2007). However, the practicality and applicability of these approaches must be examined.

Issue 9: Lack of knowledge of sensor properties and algorithms

Explanation

The properties and processing algorithms of sensors are often proprietary, classified, or under development. Therefore the modeler often does not have access to them. The information about the sensor may never be *fully* captured by the manufacturer/testing agency; there will always be some uncertainty.

Example

The Army wishes to understand the efficacy of a proposed new unattended ground sensor (UGS) for urban operations. The normal practice, at this time, is to first study the value of the sensor information in force-on-force simulations. However, because the sensor types and algorithms are unknown at the outset, this is a "chicken-before-the-egg" problem.

Possible solution(s)

- Characterize the performance using idealized information theoretic measures, such as the Neyman-Pearson criterion or Fisher information. These measures often assume the performance of the actual sensor algorithms is close to ideal. On the other hand, they do not require proprietary or classified information, and typically are computationally efficient.
- Use "proxy" sensors and algorithms (sensors and algorithms thought to be close to the actual system of interest) until more information about

the actual sensor is known. A more sophisticated approach would be to use a variety of proxy sensors.

Issue 10: Complex, dynamic, unknown scenarios in which the sensors are used

Explanation

Uncertainty of *when*, *where*, and *what kind of* predictions are required is often a key consideration; scenarios in which the sensors are used are complex and vary dynamically as the warfighter responds to the battlespace. The geometry between target and sensor is not entirely known in advance.

Examples

- IR visibility of a target from the air typically depends strongly on the elevation angle and target orientation. These angles vary, however, over the course of an engagement and are often not known in advance.
- To avoid disruptions from hostile forces, a communications network must be laid out in an ad hoc manner, with the individual devices in different positions than initially planned.

Possible solutions

- “Play” different tactical and weather scenarios as part of pre-mission training exercises.
- Perform statistical analyses on a number of likely scenarios.
- Attempt to provide useful, general information that will apply to a number of situations when specific tactical details are unknown.

Issue 11: Uncertainty of when and where predictions are required

Explanation

Missions are often planned days, weeks, or months in advance. Thus, environmental conditions at the time of mission execution may be known only in a typical or climatological sense. Military engagements are also very dynamic, with human factors and activity greatly altering the battle environment.

Examples

A brigade is to be deployed with advanced UGS systems and wants guidance on how to place the sensors. Time of deployment is weeks or months from present, so weather/terrain conditions are only known in a climatological sense. The locations of interest for sensor employment are not yet fully determined.

Possible solutions

Use climatologically based, or actual historical data, as representative atmospheric and terrain conditions for the region of operations. Identify strategically significant spatial-temporal behavior based on the climatological/historical conditions.

3 Describing Uncertainty

> "There are known knowns; there are things we know we know. We also know there are known unknowns; that is to say, we know there are some things we do not know. But there are also unknown unknowns — the ones we don't know we don't know." *Donald Rumsfeld*

Identification, quantification, and reduction of uncertainty have long been sources of practical concern and vital areas of research. Our particular problems related to signal propagation and sensor performance modeling should be considered in this broader context. That is the purpose of this brief section.

The quote at the beginning of this section exemplifies a practical perspective on uncertainty. The *known knowns* represent those aspects of a mission that are understood with certainty. The phrases *known unknowns* and *unknown unknowns* establish two different degrees of uncertainty, based on whether the uncertainty is explicitly recognized. If it is recognized, it constitutes a known unknown, and can perhaps be dealt with systematically. An unknown unknown, being unrecognized, could result in a surprise, possibly a dangerous one. Allen et al. (2006) mention the existence of *known* and *unknown uncertainties* in the context of weather and climate forecasting; these correspond to Rumsfeld's known unknowns and unknown unknowns.

The two degrees of uncertainty, known unknowns and unknown unknowns, can be associated with many of the issues described in the preceding section. For example, "Issue 2: Wave propagation sensitivity to irresolvable and highly variable environmental details" and "Issue 4: Inaccurate and missing environmental data" are usually recognized limitations in a predictive capability and hence known unknowns. "Issue 7: Imperfect physics in wave propagation and signature models" and "Issue 10: Complex, dynamic, unknown scenarios in which the sensors are used" exemplify unknown unknowns when the limitations of the modeling effort go unrecognized.

One might ask whether the fourth remaining combination of the words *known* and *unknown* not present in the Rumsfeld quote, namely *unknown knowns*, should be included among possible situations involving uncertainty. Because the overt purpose of a DST is to allow a warfighter to make expert decisions without him/her necessarily possessing extensive expertise, it is to be anticipated that the warfighter will *not* be aware of many uncertainties in the DST predictions. The modeler who created the DST has a much better appreciation for the limitations of the underlying models than does the user. In effect, the DST creator's known knowns or known unknowns are often the warfighter's unknown knowns or unknown unknowns. When key information in a decision process may exist in the form of unknown knowns (unknown to the warfighter, known to the developer), methods should be developed to quantify and communicate this information.

The discussion in the preceding paragraph anticipates the concept of risk. Although a warfighter would normally have higher priorities than becoming an expert in sensor phenomenology, he or she would normally have a strong interest in avoiding risk. *Uncertainty* and *risk* are related, but not equivalent. Risk implies a state of uncertainty that could lead to an unfavorable outcome. Uncertainty by itself is not a concern unless there is associated, significant risk. Although uncertainty is emphasized in this report, we recognize that the end value of the uncertainty models lies in recognizing, evaluating, and dealing with risk.

In the academic study of statistics, uncertainty is often classified as either *epistemological* (pertaining to knowledge) or *aleatory* (pertaining to luck). Sometimes, these are more simply called *uncertainty* and *randomness*, respectively. Issue 2 from the previous section, "Wave propagation sensitivity to irresolvable and highly variable environmental details," would normally be considered aleatory uncertainty. Practically speaking, variations in sound levels caused by turbulent eddies and other small, dynamic environmental features must be regarded as a random process. On the other hand, uncertainty in the weather for a particular time and location would normally be considered epistemological uncertainty because with more information the accuracy of the atmospheric state characterization could possibly be improved. Still, the distinctions between epistemological and aleatory uncertainty are not always clear cut; they are often more a matter of viewpoint or purpose. Regarding weather forecasts, chaos theory teaches us that the state of the atmosphere at a distant time

in the future cannot be known deterministically, because it depends on small, random perturbations to the present atmospheric state. Hence, a long-range forecast may be considered essentially random. This is the essence of "Issue 6: Random (chaotic) behavior of atmospheric and dependent processes."

Many authors and organizations have developed their own taxonomies for uncertainty. Some attempt to be comprehensive, whereas others are focused on particular applications. In modeling, it is common to distinguish *model* and *parameter* uncertainty (e.g., Frey 1998). Model uncertainty results when the real world is abstracted with a model; this process invariably involves simplifying assumptions. Parameter uncertainty relates to the values, such as measurements, empirical constants, and decision variables that are used in the model. Similarly, regarding weather forecasting, Palmer (2006) singles out *model* and *initial* uncertainty. The latter is uncertainty in the initial conditions (atmospheric observations) of the weather forecast model. Among the uncertainty issues raised earlier, initial uncertainty is most closely matched by "Issue 4: Inaccurate and missing environmental data." Model uncertainty is present in many issues, including 5, 7, and 9–11.

Lastly, we mention the taxonomy of Ayyub and Chao (1998) who consider uncertainty in the context of civil engineering. Their classification scheme, at its initial level, pertains to the *origin* of the uncertainty: *ambiguity*, *vagueness*, or *information conflicts*. Ambiguity refers to noncognitive (definable) issues such as physical randomness, limited sampling of a process, lack of knowledge, and modeling uncertainty. Issues 1–3 and 6–7 fall primarily into the category of ambiguity uncertainties. Vagueness relates to the inability to precisely define parameters and judgments (such as system failure or survival) and describe interrelationships between elements of a complex system. Issues 5 and 8–11 deal primarily with vagueness.

4 Probabilistic Framework for a DST

We now consider formulation of a probabilistic framework for sensor performance prediction including uncertainty. Ideally, the framework should be fairly simple, and yet accommodate the many sources for uncertainty mentioned in the preceding section. It should also be compatible with established approaches to dealing with uncertainty, such as ensemble weather forecasting, and leverage previous DST calculations based on a crisp specification of the environmental and tactical scenarios.

This section is structured as follows. We first analyze the important tasks performed by a DST, so we might better understand where uncertainty can occur. We then develop a probabilistic formulation for these tasks. Lastly, we consider a computational architecture for a DST with uncertainty.

What does a DST do?

Before attempting to formulate a probabilistic framework, it is helpful to take a step back and describe the general functions of a DST. We consider these to be the following:

- *Step 1: information gathering* and construction of the tactical and environmental (atmosphere and terrain) scenario,
- *Step 2: translation* of the scenario information to parameters needed by the signal and noise prediction models,
- *Step 3: signal and noise prediction* models (source characteristics, propagation through the environment, and sensor processing),
- *Step 4:* calculation of *sensor performance metrics* (e.g., probability of detection or classification, or source localization accuracy), and
- *Step 5: display* of and interaction with the information (graphical interface).

Figure 2 conceptually shows the first four steps of this process as implemented in a crisp DST (one in which the environmental information and tactical scenario are assumed to be exactly known), such as in the Sensor Performance Evaluator for Battlefield Environments (SPEBE) (Wilson and Szeto 2000; Wilson 2006).

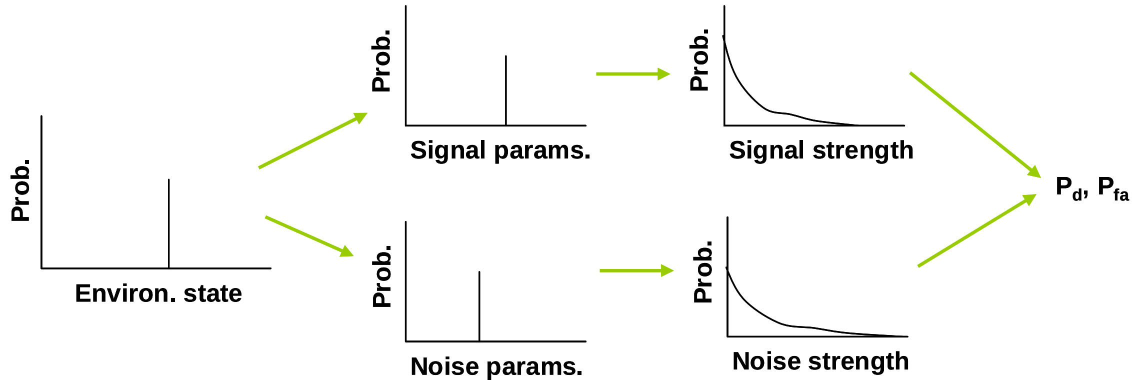

Figure 2. Probabilistic model underlying a crisp (no uncertainty) DST calculation. A single environmental (atmospheric and terrain) state is specified, and assumed to be exact. The environmental variables lead to deterministic input parameters to the signal and noise predictions. Based on these parameters, probability distributions are calculated for the signal and noise at the sensor. From these, metrics for the sensor performance (such as the probabilities of detection, P_d, and false alarm, P_{fa}) follow.

Although the calculation Step 3 is the heart of the DST, the other steps are all very challenging and important. In SPEBE, for example, Step 2 involves turbulence similarity theory modeling to create local, detailed atmospheric wind and temperature profiles from coarsely gridded weather forecasts; complicated models for the acoustical properties of the ground, as determined by the soil permeability and porosity, are also used.

In a typical scenario, the environmental information supplied to the predictions (Step 1) is derived from multiple sources. These include *externally* supplied environmental information that is "pushed" to the local analyst from existing databases or higher command echelons. This information may include the large-scale weather forecast, a digital terrain elevation map, ground surface properties, and cultural entities in the region of interest. There may also be *locally* supplied environmental information supplied by echelon-level sensors and field observations or as otherwise necessary to complete the inputs needed for the sensor prediction models. Our previous experience indicates that the externally supplied information is often insufficient for the predictive models and therefore the analyst often must fill in some details to perform accurate calculations with the physical models. This information may supplement, constrain, or even contradict, the external information. For example, the large-scale forecast area includes a variety of terrain types (i.e., fields, forest, suburban, and urban) but the local analyst knows the terrain in which the operation will take place. Mathematically, we designate the parameters representing environmental information from N data sources as $\psi_1, \psi_2, \ldots, \psi_N$.

One might reasonably ask why we do not distinguish between atmospheric and terrain information, at least in a notational sense. Are not these data independent (e.g., a weather forecast and a digital elevation map)? There are actually many situations where the terrain and atmosphere have important feedbacks. For example, precipitation alters the soil properties. By combining the atmospheric and terrain parameters, we have made a notational decision consistent with an interacting atmosphere and terrain, although in practice the data may be obtained from different sources.

In addition to the environmental parameters, information must be gathered regarding the tactical scenario, by which we mean here the types of targets and sensors, their positions, orientations, etc. This information is indicated here with the symbol ξ. Conceivably, there could exist situations where the tactical parameters depend on the environmental state, although we will not consider that complication in the present model. One might now ask whether, analogously to the environmental parameters, we should consider multiple information sources for the tactical parameters. In our experience, information on the tactical parameters usually comes from a single data source, e.g., a command and control system or through manual entry by the user. Hence, we do not consider multiple sources, although it would be rather simple to add such a complication.

Taken together, the environmental information $\psi_1, \ldots, \psi_N$ and tactical scenario information ξ provide the parameters χ needed by the signal generation (target signature), transmission (propagation), and reception (sensor) prediction models. (In the following, we will call these several models the *signature propagation* model for brevity.) This is Step 2 from the preceding description. The parameters χ are to be determined in a manner that is statistically consistent with $\psi_1, \ldots, \psi_N$ and ξ; typically, only a subset of the supplied information will be needed. For example, an acoustic prediction will need information on the atmospheric wind field, but fog is usually unimportant; an IR prediction will depend on fog but not as strongly on wind.

Next, we feed the information χ into the signature propagation model to determine the signal and noise *features* at the sensor, *s* and *n*, respectively (Step 3). These features may represent different modalities (acoustic, seismic, visible, IR, and/or electromagnetic) as well as different frequency bands and statistics for these modalities. Due to unresolvable phenomena such as turbulence, vegetation, and small buildings, the signal and noise

are generally random variables. When modeled probabilistically, we describe them with probability density functions (pdfs). The pdfs for *s* and *n* depend on sets of parameters θ_s and θ_n. For Gaussian pdfs, for example, these parameters would consist of means and standard deviations of the signal and noise features.

Step 4, the formation of metrics, involves processing the signal and noise information. For example, a probability of detection or classification may be calculated from θ_s and θ_n. Once this processing has been performed, the information can be visualized (Step 5).

In summary, we have the following symbols, each of which actually represents a (perhaps very large) set of parameters:

- $\psi_1, \ldots, \psi_N =$ atmospheric and terrain parameters
- $\xi =$ tactical scenario parameters
- $\chi =$ parameters for the signal/transmission/sensor prediction models
- $\theta_s =$ pdf parameterization for the signal features
- $\theta_n =$ pdf parameterization for the noise features

An "object-oriented" discussion

In this section, we outline how the functionality for a crisp DST, as laid out in the previous section, might be implemented in software. Later, we consider how this functionality could be generalized to include uncertainty. The discussion is based on a modern, "object-oriented" view to programming, as exemplified by languages such as Java and C#. Discussing the problem in this manner helps to organize one's thoughts and describe them in a manner that can be more readily programmed. For the uninitiated, an *object* may represent a real-world entity or more abstract concept. Objects have characteristics, which may alternatively be called properties, data, or (in Java) fields. They also have functionality and perform tasks, which are usually called *methods*. Particular objects are *instances* of *classes*. For example, one might consider all helicopters to comprise a class. A particular helicopter would be an instance of this class. The object characteristics are the design of the helicopter and the instruments on it. The functionality consists of flying, firing weapons, etc.

Regarding the discussion in the previous section, we might consider the environmental and tactical state parameters ($\psi_1, \ldots, \psi_N$ and ξ) to be an in-

stance, of, say, an environmental/tactical data model class. For brevity, we will call such an instance a *scenario*. The class must define the particular properties that represent a scenario, such as the atmospheric wind and temperature, the ground permeability and albedo, the target and sensor types and positions, etc. In actual problems the environmental/tactical data model can be expected to be quite complicated and actually collect many different objects. Conceivably, different environmental/tactical data classes could be developed, depending on the level of sophistication and the information that is available or desired. In Java, we might thus develop a "parent class" environmental/tactical data model (a general, high-level description) and "subclasses" for more particular situations. Or, one might use a Java interface specification, and then develop subclasses that implement this interface.

The information gathering step (Step 1) can be viewed as constructing the scenarios. The construction could involve many different data resources, such as weather forecasts, digital terrain models, and inputs directly supplied by the user. Conceivably, many alternative methods could be developed to construct scenarios, involving processes such as retrieving data over the Internet, reading data from media, and allowing users to enter data interactively. The processes of selecting data resources and entering data would likely involve a graphical user interface (GUI).

Next we must translate the scenario objects to parameters needed to run our signature propagation predictions (Step 2). These parameters can be considered a new type of object. We describe a method for converting scenarios to prediction model parameters as a *scenario translator*. Different subclasses of prediction model parameters could be defined as needed for different prediction models. The prediction models could be implemented as methods for these subclasses. Indirectly, the prediction models thus operate on different environmental/tactical data model classes. The scenario translator serves as the intermediary.

Next we consider the signal and noise features, as parameterized by θ_s and θ_n. A signature propagation model can be regarded as a method that produces θ_s and θ_n. This is Step 3. It is expected that different methods would be used to produce different kinds of features. For example, an acoustic prediction model would produce parameters related to acoustic features.

Step 4 would consist of a set of methods that operate on signal and noise feature parameterizations to produce sensor metrics of interest, such as probability of detection or classification. Step 5 displays the results of these methods.

If good, modular coding practices are followed, most of the object-oriented code should be portable; that is, it should be able to run under different GUIs and geospatial information systems (GIS) with little or no change. The information gathering in Step 1 would initially involve some GUI-specific operations to gather the data, but then the construction of scenarios from this information should rely on portable code. Once Step 1 has been finished, Steps 2, 3, and 4 should proceed entirely with portable code. Step 5, of course, is GUI specific.

Bayesian probabilistic formulation

We now consider incorporation of uncertainty into the DST predictions. Many different approaches can be contemplated, including traditional probability-based statistics, or newer approaches such as fuzzy logic and belief functions. Here, we develop a formulation based on Bayesian probabilistic concepts, which means that probabilities are conditioned on prior knowledge. The scheme is loosely inspired by Maurer et al.'s (2006) Bayesian formulation for ballistic missile signatures.

The information gathering, Step 1, amounts to determining pdfs for $\psi_1, \ldots, \psi_N$ and ξ. In the most general sense, these parameters will possess a joint pdf $p\left(\psi_1, \ldots, \psi_N, \xi\right)$. However, as mentioned previously, we assume that the environmental and tactical parameters are independent, in which case (by definition) $p\left(\psi_1, \ldots, \psi_N, \xi\right) = p\left(\psi_1, \ldots, \psi_N\right) p(\xi)$. Here $p\left(\psi_1, \ldots, \psi_N\right)$ describes the ubiquitous uncertainty in the environmental state. A distribution $p(\xi)$ might be important, say, when there is uncertainty in the altitude of an aircraft.

The translation, Step 2, involves mapping the information in $\psi_1, \ldots, \psi_N$ and ξ to the signature propagation model parameters χ. Probabilistically, this mapping is indicated by the likelihood function $p(\chi \mid \psi_1, \psi_2, \ldots, \psi_N, \xi)$, where the vertical bar indicates that the variable on the left is conditioned to the values on the right. According to the definition of conditional probability,

$$p(\chi \mid \psi_1, \ldots, \psi_N, \xi) = \frac{p(\chi, \psi_1, \ldots, \psi_N, \xi)}{p(\psi_1, \ldots, \psi_N, \xi)}. \tag{1}$$

The marginal pdf $p(\chi)$ is determined by integrating $p(\chi, \psi_1, \ldots, \psi_N, \xi)$ over $\psi_1, \ldots, \psi_N$ and ξ. The result is

$$p(\chi) = \iint p(\chi \mid \psi_1, \ldots, \psi_N, \xi)\, p(\psi_1, \ldots, \psi_N, \xi)\, d\boldsymbol{\psi}\, d\xi, \tag{2}$$

where the boldface $\boldsymbol{\psi}$ indicates a vector containing $\psi_1, \ldots, \psi_N$. The prediction, Step 3, can be considered a mapping of the model prediction parameters to the pdf parameters for the signal and noise features. When this mapping involves uncertainty, we represent it notationally as the likelihood function $p(\theta_s, \theta_n \mid \chi)$. By definition,

$$p(\theta_s, \theta_n \mid \chi) = \frac{p(\theta_s, \theta_n, \chi)}{p(\chi)}. \tag{3}$$

An example situation where it is important to implement a distribution $p(\chi)$ is when the ground surface may consist of grass or asphalt, in which case the sound propagation characteristics will be very different. Another observation to make regarding Step 3 is that, since the propagation model parameters χ contain all the information from $\psi_1, \ldots, \psi_N$ and ξ affecting the signal and noise distributions, $p(\theta_s, \theta_n \mid \chi) = p(\theta_s, \theta_n \mid \psi_1, \ldots, \psi_N, \xi)$. From Equation 3 and then substituting with Equation 2 we have

$$p(\theta_s, \theta_n) = \int p(\theta_s, \theta_n \mid \chi)\, p(\chi)\, d\chi = \int p(\theta_s, \theta_n \mid \chi) \left[\iint p(\chi \mid \psi_1, \ldots, \psi_N, \xi)\, p(\psi_1, \ldots, \psi_N, \xi)\, d\boldsymbol{\psi}\, d\xi \right] d\chi. \tag{4}$$

This parameterization of the signal and noise pdf might be needed, for example, to perform a simulation. For a DST, in the end we would typically like to know the expected value of functions that depend on θ_s and θ_n, such as the probabilities of detection and false alarm. From Equation 4 we have

$$\left\langle f(\theta_s, \theta_n) \right\rangle = \iint f(\theta_s, \theta_n) \left\{ \int p(\theta_s, \theta_n \mid \chi) \left[\iint p(\chi \mid \psi_1, \ldots, \psi_N, \xi)\, p(\psi_1, \ldots, \psi_N, \xi)\, d\boldsymbol{\psi}\, d\xi \right] d\chi \right\} d\theta_s\, d\theta_n. \tag{5}$$

Alternatively we might wish to determine the joint pdf of *s* and *n* themselves, $p(s,n)$. Assuming we have a model for $p(s,n \mid \theta_s,\theta_n)$, as might be known for example from a random scattering theory, Equation 4 and further application of Bayes' theorem leads to

$$p(s,n) = \iint p(s,n \mid \theta_s,\theta_n)\left\{\int p(\theta_s,\theta_n \mid \chi)\left[\iint p(\chi \mid \psi_1,\ldots,\psi_N,\xi)\, p(\psi_1,\ldots,\psi_N,\xi)\, d\boldsymbol{\psi}\, d\xi\right] d\chi\right\} d\theta_s\, d\theta_n. \tag{6}$$

If the signal and noise are independent, then $p(s,n) = p(s)\,p(n)$. Another point to make regarding Equations 5 and 6 is that there is no causative interaction between the individual signal and noise features; for example, an IR feature does not alter an acoustic feature (although they may be affected by common environmental and tactical parameters). Hence, their pdfs may be calculated independently. If the probabilistic models for the environment and tactical scenario are discrete (but the signal and noise pdfs still continuous), we have, instead of Equation 6,

$$p(s,n) = \sum_{\theta_s}\sum_{\theta_n} p(s,n \mid \theta_s,\theta_n) \sum_{\chi} P(\theta_s,\theta_n \mid \chi) \sum_{\psi_1}\cdots\sum_{\psi_N}\sum_{\xi} P(\chi \mid \psi_1,\ldots,\psi_N,\xi)\, P(\psi_1,\ldots,\psi_N,\xi). \tag{7}$$

Here, the uppercase *P* indicates the probability of a particular value of its argument.

Equations 5–7 are various forms of our conceptual model for predictive uncertainty. Given that these results are six-dimensional integrals (or summations), solutions could be very computationally intensive. Instead of performing the integrations literally, it is probably necessary to develop a sampling strategy (such as Monte Carlo or Latin hypercube sampling). Also, only the most important sources of uncertainty should be included. This will become more apparent in the next section, which considers some illustrative applications. Five likelihood functions and pdfs are required, which to recapitulate are:

- $p(s,n \mid \theta_s,\theta_n)$, the joint likelihood function for the signal and noise variations based on parameterizations of their pdfs. Alternatively, the problem may require a deterministic function $f(\theta_s,\theta_n)$. The capability to perform this calculation may often be extracted from an existing DST.

- $p(\theta_s, \theta_n \mid \chi)$, the joint likelihood function for the signal and noise pdf parameters corresponding to a particular set of prediction models. The capability to perform this calculation may often be extracted from an existing DST.
- $p(\chi \mid \psi_1, \ldots, \psi_N, \xi)$, the likelihood function for the signature propagation model parameters given the large-scale weather forecast and terrain information, locally supplied weather and terrain information, and tactical information. In practice, there is not much guidance for this likelihood function; it may be challenging to model it well.
- $p(\psi_1, \ldots, \psi_N)$, the pdf of a particular weather and terrain state. This could be available from an ensemble forecast system or a model that generates sets of typical atmospheric conditions from climatology as well as terrain databases.
- $p(\xi)$, the pdf of a particular tactical scenario. This information would normally be supplied by the user.

Figure 3 conceptually shows the processing for a DST that accommodates uncertainty. The pdfs for the input parameters [$p(\psi_1, \ldots, \psi_N)$ and $p(\xi)$] are represented in the figure as discrete (initial stage, starting at the left of the figure). These could be generated, for example, by an ensemble weather forecasting system and a fixed number of plausible tactical scenarios. For each member of the ensemble, a range of signal and noise pdf parameters is produced (second stage in the figure, i.e., Equation 2). Next, distributions for the signal and noise features are produced (third stage in the figure, Equation 4). Then, sensor performance metrics such as probabilities of detection and false alarm can be determined.

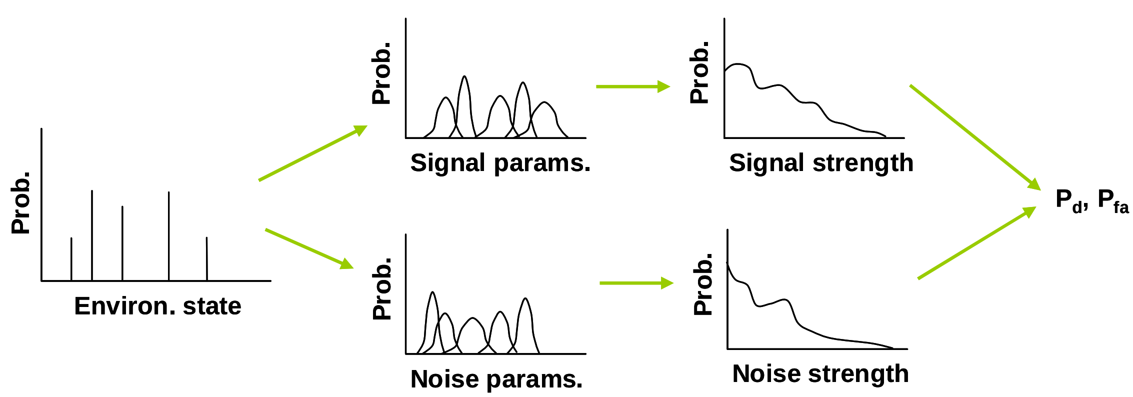

Figure 3. Probabilistic model underlying a DST calculation with uncertainty. Multiple environmental states, which may have differing probabilities, are supplied. For each environmental state, a range of input parameters is determined for the signal and noise predictions. Based on these distributed parameters, probability distributions are calculated for the signal and noise at the sensor. From these, metrics for the sensor performance follow.

Implementation architecture

The probabilistic model in the previous section is general enough to accommodate a great variety of implementations. But it still does not provide a practical prescription for including uncertainty in DSTs. The particular approach one chooses should be guided by computational feasibility and compatibility with current approaches for dealing with environmental uncertainty.

Since ensemble forecasting has become a widespread practice in weather forecasting, we anticipate leveraging this approach to dealing with uncertainty in the atmospheric state. In this manner, we directly address "Issue 5: Complex interrelationships among systems of environmental variables," and "Issue 6: Random (chaotic) behavior of atmospheric and dependent processes." The ensemble forecasts describe the atmosphere with a number of discrete set of states, which may be equally likely or assigned differing probabilities. In the notation of the preceding section, $p(\psi_1, \ldots, \psi_N)$ becomes the discrete distribution $P(\psi_1, \ldots, \psi_N)$. It is then natural to extend this approach to include tactical scenarios; that is, we consider joint, discrete pdfs $P(\psi_1, \ldots, \psi_N, \xi)$. This allows Issues 10 and 11, which deal with uncertainty in the tactical scenario, to be addressed.

The likelihood function $p(\chi \mid \psi_1, \ldots, \psi_N, \xi)$ is the appropriate point at which to introduce uncertainties affecting the signature propagation model as due to incomplete knowledge of the environment and tactical scenario. This allows "Issue 2: Wave propagation sensitivity to irresolvable and highly variable environmental details" and "Issue 4: Inaccurate and missing environmental data" to be addressed. Because, in general, the signature propagation model is a crisp formulation (i.e., it maps a single set of values for χ to a single set of values for θ_s and θ_n), we also perform this stage of the calculation using a discrete probability model. Given a particular parameter range for the elements of χ, we can use a Monte Carlo or Latin hypercube strategy to generate random samples. This is illustrated in Section 5.

If there are uncertainties in the predictions of the propagation model (i.e., Issue 7), the appropriate place to introduce them would be $p(\theta_s, \theta_n \mid \chi)$. Although this may be appropriate in many situations, this work emphasizes uncertainty in signature propagation model parameters, rather than the

model itself. Hence, no uncertainty is introduced at this stage. In effect, $p(\theta_s, \theta_n | \chi)$ is assumed to be crisp, so that the summation over χ vanishes from Equation 7 and we have

$$p(s,n) = \sum_{\theta_s}\sum_{\theta_n} p\left[s,n | \theta_s(\chi), \theta_n(\chi)\right] \sum_{\psi_1} \cdots \sum_{\psi_N}\sum_{\xi} P(\chi | \psi_1, \ldots, \psi_N, \xi) P(\psi_1, \ldots, \psi_N, \xi). \quad (8)$$

Correct application of this result will properly include correlated sensor observations (Issue 8).

Figure 4 shows a possible architecture for implementing calculations based on Equation 8. The five numbers in circles correspond to the five DST steps described earlier. Initially, the user interactively builds an ensemble of environmental/tactical scenarios. This step involves retrieval of information from various external databases containing atmospheric and

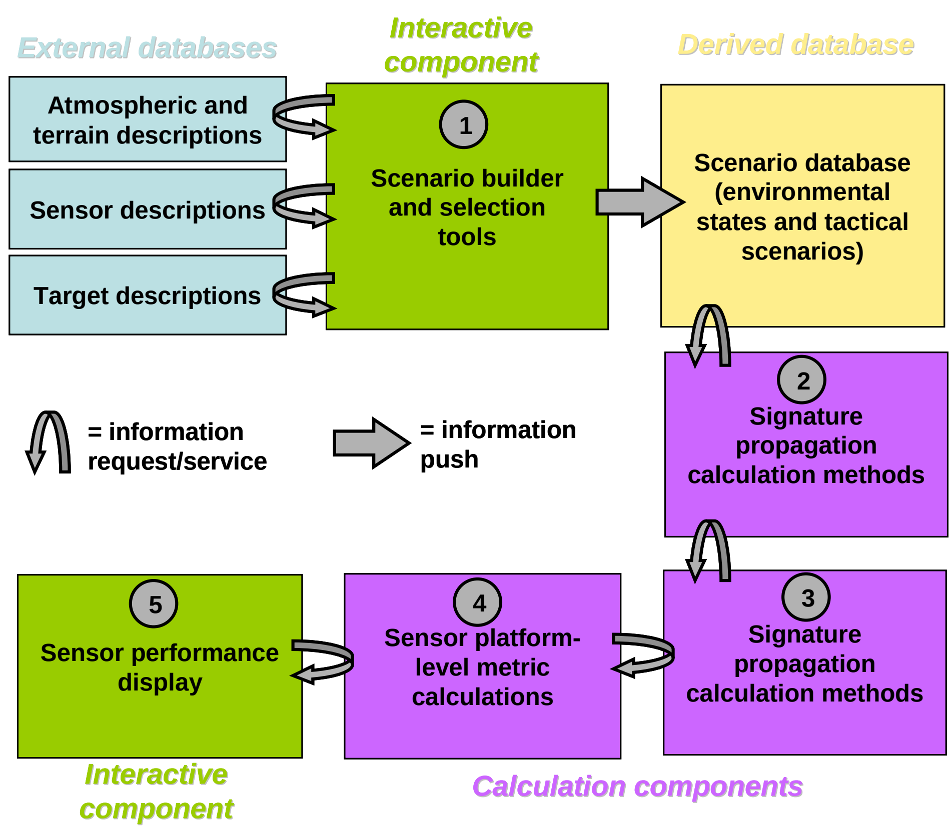

Figure 4. Software architecture for a DST that may include uncertainty in the sensor performance assessments. Initially, scenarios (environmental and tactical information) are built from externally supplied information and stored in a database. When the user requests a sensor performance display, a series of "reachbacks" are performed to obtain the necessary scenario information and apply it to a sequence of calculations.

terrain data, and information on sensors and targets. The constructed scenarios are pushed to a local database. Next, when the user requests a sensor-performance display, there is a sequence of “reachback” events. Eventually, data are pulled from the scenario database to perform a signature propagation calculation. Each scenario is normally pulled, and multiple variations of the signature propagation model parameters are generated from each scenario to account for uncertainty. The signature propagation and performance metric calculations are then performed and finally displayed.

5 Example Scenarios

Sensor selection from multiple modalities

Let us first consider an example involving multiple sensor modalities. The example, although highly idealized, illustrates how, even in rather simple scenarios, uncertainty in sensor performance can play an important role in determining which sensors or combination thereof are best suited to a particular mission. Although we call the three sensor modalities passive IR, acoustic, and seismic, the postulated performance characteristics are not based on any actual sensor. The target is also a generic one; that is, its properties do not correspond to any actual target.

Initially, we consider daytime and nighttime cases without any uncertainty in sensors' performance. Both cases involve mostly clear conditions, with sunshine during the day and a moderate, ground-based temperature inversion at night. The signal and noise are assumed to have Gaussian pdfs, with means and standard deviations for the various cases shown in Table 1. In the notation introduced earlier, the parameters in this table constitute θ_s and θ_n. All values are normalized by the mean of the noise at night. The rationale behind the parameter values is the following. For the acoustic sensor, we assume the signal is stronger at night than during the daytime, due to downward refraction of the sound at night by temperature gradients. Acoustic noise increases during the daytime due to wind. The standard deviation of the acoustic signal and noise is always high (half the mean). The seismic signal and noise are assumed to be the same, day or night. The passive IR sensor is assumed to have a strong signal, relative to the noise, at night. The signal at night is also assumed to be very steady. During the daytime, the IR signal and noise are both high, but also quite variable.

For each of these cases, we calculate receiver operating characteristic (ROC) curves using the methods described in the Appendix. The ROC curves for the three modalities together are based on the assumption that a false alarm occurs if any of the sensors reports a false alarm, and a detec-

Table 1. Signal and noise parameters for illustrative scenario involving multiple sensor modalities.

Modality, Condition	Mean, signal (μ_s)	Standard Deviation, signal (σ_s)	Mean, noise (μ_n)	Standard Deviation, noise (σ_n)
Passive IR, daytime	5	2.5	3	1.5
Passive IR, nighttime	3	0.1	1	0.5
Acoustic, daytime	2	1	2	1
Acoustic, nighttime	3	1.5	1	0.5
Seismic, daytime	2	0.5	1	0.5
Seismic, nighttime	2	0.5	1	0.5

tion occurs if any of the sensors reports a detection. Results for the daytime case (Figure 5, left) indicate that (lacking any uncertainty in the sensors' performance) the seismic sensor will be the most valuable, followed by the passive IR. The acoustic sensor performs comparatively poorly due to the low signal-to-noise ratio. Overall, the passive IR and seismic sensors combine to provide better detection than either alone. (The ROC curve for the combination of all the sensors is based on the premise that detection occurs if *any* of the sensors report a detection. This is not necessarily the optimal assumption.) For the nighttime case without uncertainty (Figure 6, left), the passive IR sensor is best. The acoustic sensor is next best and moderately enhances the probability of detection in comparison to the IR sensor alone, particularly for low false alarm probabilities. The seismic sensor is relatively unimportant.

Next we introduce uncertainty into the daytime case. Specifically, the local vegetative cover is uncertain. The probability of being in an uncultivated field with high vegetation, which partially obscures the target, is taken to be 70%. There is a 30% chance the target is in a cultivated field with low vegetation and hence less obscuration. In the notation of the previous section, $P(\psi = \psi_h) = 0.7$, where ψ_h is the set of terrain parameters describing

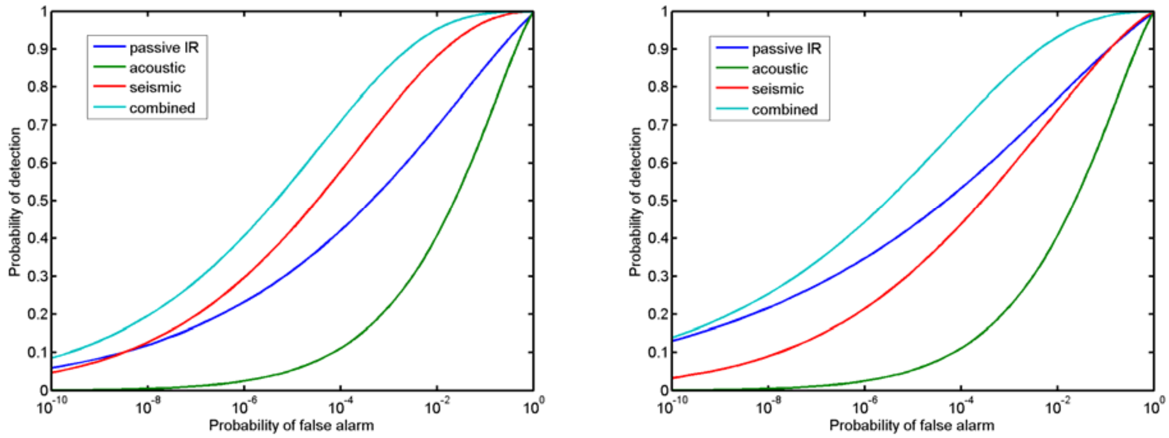

Figure 5. ROC curves for the daytime case. Calculation on the left assumes with certainty that the sensors are in an open field. The calculation on the right is based on a 70% chance of the sensors being in a field with high vegetation, and 30% chance of the sensors being in a field with low vegetation.

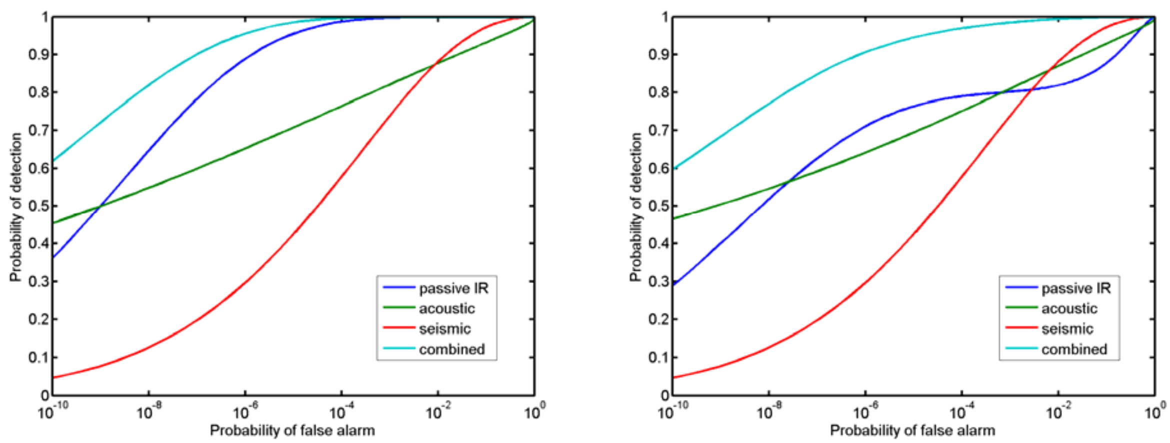

Figure 6. ROC curves for the nighttime case. Calculation on the left assumes with certainty that there is a moderate temperature inversion. The calculation on the right is based on a 60% chance of a moderate temperature inversion, a 20% chance of a weak temperature inversion, and a 20% chance of a strong temperature inversion with fog.

the partially obscured case, and $P(\psi = \psi_l) = 0.7$, where ψ_l is the set of terrain parameters for the less obscured case. The mapping $p(\theta_s, \theta_n \mid \psi, \xi)$ is assumed to remain crisp. For the less obscured case, the mean of the IR signal is increased from 5 to 8, to represent improved target contrast. The mean seismic noise is doubled in the cultivated field to represent increased vibrations from nearby cultural activity. Now (Figure 5, right), the passive IR sensor typically provides more useful data, although overall the ROC curve is not strongly affected.

Uncertainty is introduced into the nighttime case regarding the strength of the temperature inversion layer (the vertical temperature gradient). The probability of a moderate, ground-based temperature inversion is taken as 60%. The probability of a weak temperature inversion (as typically occurs at night after an intermittent episode of turbulent mixing) is taken to be 20%. For this situation, the acoustic signal mean is reduced from 3 to 2, to represent weaker downward refraction. The probability of a strong temperature inversion (as typically occurs at night before an intermittent episode of turbulent mixing) coupled with fog is also taken to be 20%. For this situation, the acoustic signal mean is increased to 5, to represent stronger downward refraction. The IR signal mean is reduced to 0.5 due to obstruction by fog. With this combination of uncertainty, the IR and acoustic sensors are both necessary to maintain a high probability of detection (Figure 6, right).

Finally we consider the case where the signal and noise are correlated. Such correlations can occur when the signal and noise are affected by the same phenomenon; for example, infrared detection in fog. In Figure 7, we show the nighttime case with uncertainty as described above, but with correlation coefficient $\rho_{s,n} = -0.5$ (left) and $\rho_{s,n} = 0.5$ (right). As is most clearly evident in the seismic ROC curves, negative signal-noise correlation creates a sharper transition between regimes of low and high probability of detection, whereas positive correlation creates a smoother transition. Similar effects can be shown when a nonzero correlation is introduced into the other cases considered earlier.

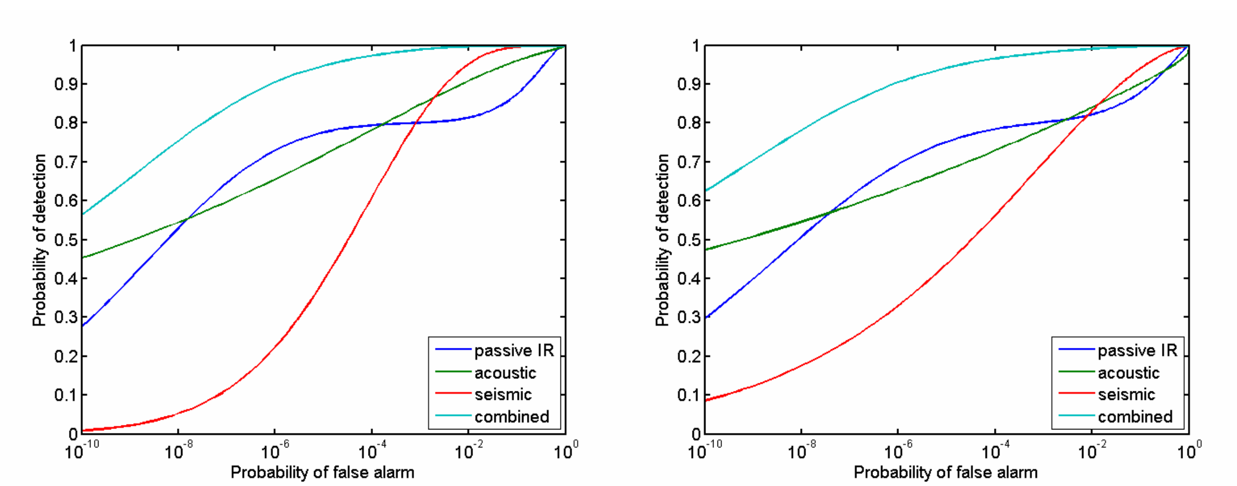

Figure 7. ROC curves for the nighttime case with uncertainty and $\rho_{s,n} = -0.5$ (left) and $\rho_{s,n} = 0.5$ (right). The other parameters are the same as the ones for the plot on the right of Figure 6.

Uncertain weather and terrain in acoustical predictions

In this section, we consider the effect of uncertainty in the context of realistic models for sound propagation, including terrain and weather effects. The calculations were performed with SPEBE (Wilson and Szeto 2000; Wilson 2006). The terrain elevation model (shown in Figure 8, left) is a random, fractal one, with characteristic length 1 km and root-mean-square height variation 200 m. The domain is 5 km on a side. The source is harmonic with frequency of 100 Hz and sound level 120 dB referenced to 20 μPa. The microphone characteristics are based on a typical, piezoelectric design. A constant false-alarm probability $P_{fa} = 10^{-6}$ is used to set the detector threshold. Meteorological conditions are neutral (temperature decreasing with height according to the adiabatic lapse rate) as is characteristic of cloudy, windy conditions. A parabolic equation solver is used to calculate the atmospheric effects on sound propagation.

Five parameters in the calculation were varied to reflect uncertainty in the model inputs. These are the source height, the ambient noise background, the static flow resistivity of the ground (proportional to the inverse permeability; this parameter affects the acoustic reflectivity of the ground), the wind speed, and the wind direction. Specifically, the source height varies uniformly from 40 to 200 m above ground level. The mean source height, 120 m, is taken to be the baseline value, i.e., the value when parametric uncertainty is neglected. The ambient noise background varies uniformly between the "quiet" (66 dB) and "loud" (86 dB) curves appearing in Becker and Güdesen (2000), with the "medium" (76 dB) curve as the baseline. This corresponds to a total dynamic range of 20 dB. The *logarithm* of the static flow resistivity varies uniformly from log(50,000 Pa s m^{-2}) to log(300,000 Pa s m^{-2}). This range corresponds to conditions from soft to compacted soil. The baseline value is 123,000 Pa s m^{-2}. The wind speed varies uniformly from 5 to 10 m s^{-1}, with baseline 7.5 m s^{-1}. Note that the calculations include a wind noise model, i.e., higher wind speed increases the noise at the microphone. The wind direction varies uniformly -45° to 45°, with mean 0°. (A Cartesian convention is used for the wind direction, i.e., 0° is *to the east*. Note that this convention differs from the usual meteorological convention, where 0° is *from the north*.) The probability of detection calculation from the baseline values is shown in Figure 8 (right).

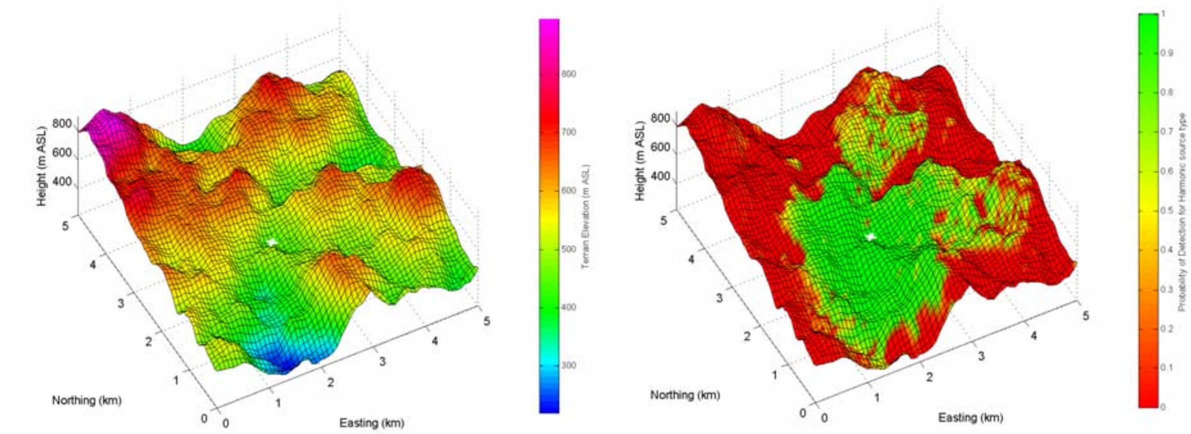

Figure 8. Synthesized terrain elevation model for the acoustic detection calculations (left), and baseline detection calculation, i.e., the calculation lacking parametric uncertainty (right). Green indicates high probability of detection; red indicates low probability. The source position is indicated by an “x”. (The sensor position is varied across the terrain.)

Figure 9 indicates the sensitivities of the predictions to the various parameters. The top row is the calculation for the parameter at its *minimum* value, with all other parameters held to their baseline values. The bottom row is the calculation for the parameter at its *maximum* value, with all other parameters held to their baseline values. Variations in the source height have the maximum impact: as the source is raised, the sound propagates more readily over the hills. The effect of the wind speed appears to be smallest, although it should be kept in mind that Figure 9 does not depict how combinations of variations in the parameters may affect the results.

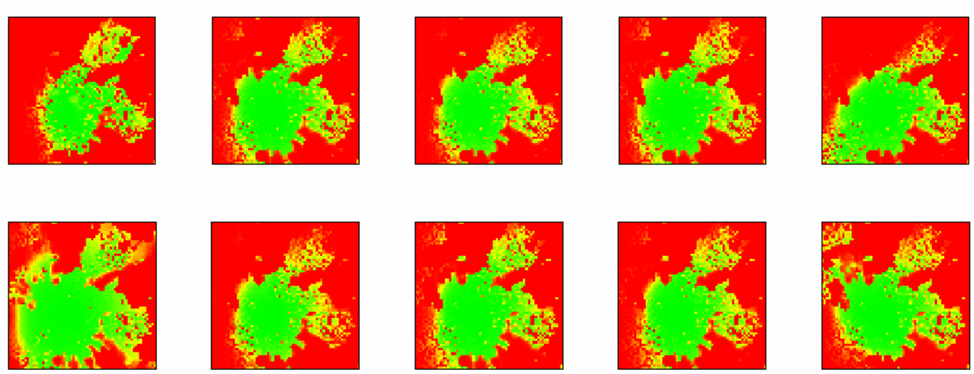

Figure 9. Sensitivity of predictions to parameter variations. For each column, only one parameter is varied; all others are fixed to their baseline values. The columns show, respectively, the effect of variations in the source height, noise background, static flow resistivity, wind speed, and wind direction. Top row is for the parameter at the minimum value of its range; bottom row is at the maximum value.

To simulate the effect of parametric uncertainty, random parameter combinations were generated by LHS. (Pettit and Wilson [2007] provide a more detailed discussion of the application of LHS to sound propagation.) A total of 16 random cases were created; the corresponding calculation results are shown in Figure 10. The most favorable case, in the sense of highest spatially averaged probability of detection, is 7 (counting across the rows and then down the columns). This corresponds to source height 167 m, noise background 71.3 dB, flow resistivity 204,000 Pa s m^{-2}, wind speed 5.27 m s^{-1}, and wind direction 44.6°. The least favorable case is 13, which corresponds to source height 93.1 m, noise background 81.6 dB, flow resistivity 67,000 Pa s m^{-2}, wind speed 9.11 m s^{-1}, and wind direction -27.1°. In general, the combination of high source height and flow resistivity, and low noise background and wind speed, are most favorable to detection.

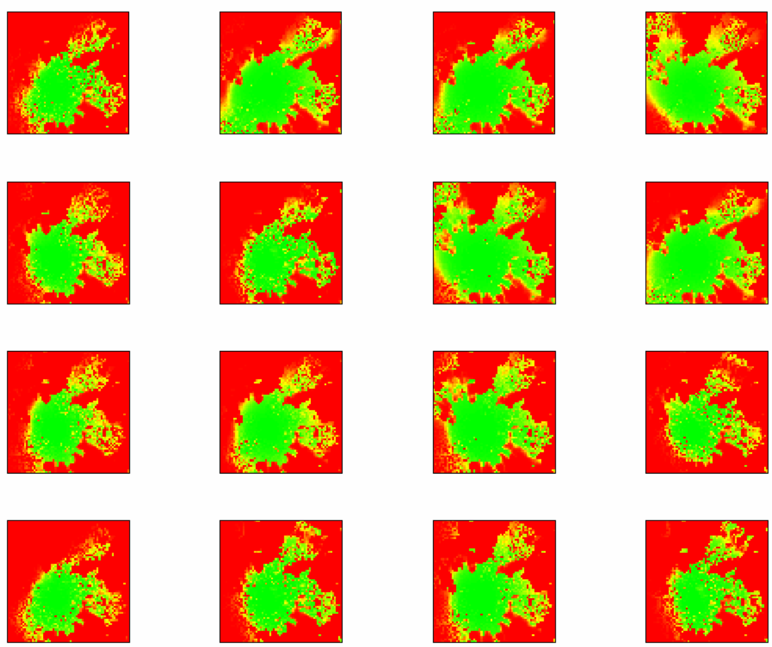

Figure 10. Probability of detection calculations based on 16 random combinations of source height, noise background, ground permeability, wind speed, and wind direction. The individual plots correspond to an overhead view of the probability of detection as in Figure 5 (right), with the horizontal coordinate being the easting and vertical being the northing.

Mean probabilities of detection, based on the first four samples and on all 16 samples, are shown in Figure 11. In comparison to Figure 8 (right), we see that inclusion of uncertainty significantly smoothes the appearance of the detection footprint. However, many of the features related to terrain elevation variations (enhancement of detection on hillsides facing the source and diminishment on hillsides facing away) are robust. Detection is

also consistently better when the wind is blowing from the source to the sensor.

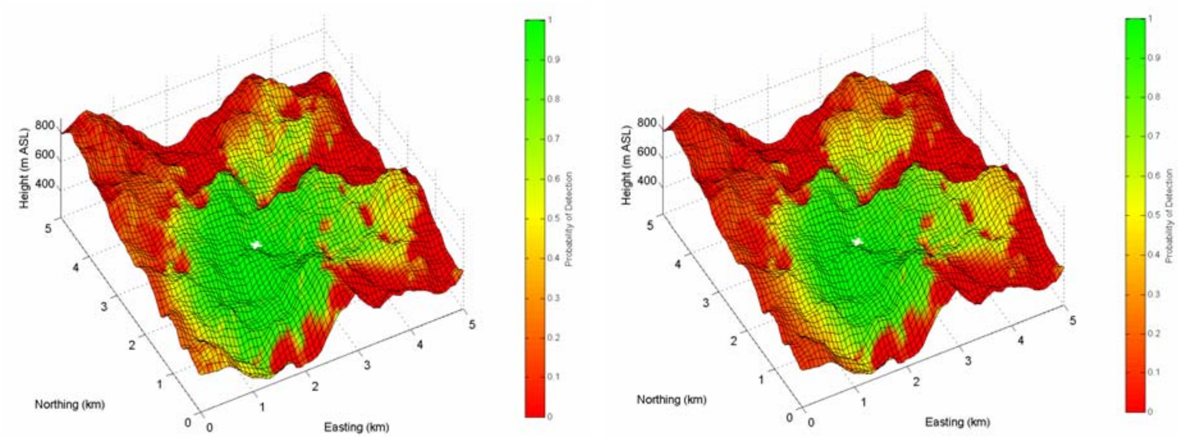

Figure 11. Mean probability of detection. Left is derived from the first four Latin hypercube samples; right is derived from all 16 samples.

6 Conclusion

Predictions from complex models are much more valuable to decision makers when there is a good understanding of the predictive skill. However, most current decision support tools for recommending sensor mixes and placements do not provide such an understanding. There are a variety of reasons for this state of affairs. First, the quality of the inputs to the DSTs is often in itself poorly understood. Second, the prediction models are often so complex that mapping of uncertainties in the model inputs to their impact on the model outputs is extremely challenging, if not impossible. Third, most developers of DSTs are more comfortable with programming and physical modeling than with statistical assessments of uncertainty and the subsequent decision-making process. With regard to this last point, it needs to be recognized that more complex, computationally intensive physical models may not significantly improve overall predictive skill when other sources of error, such as errors in model inputs, are substantial.

In this report, we have started to address the need for a balanced examination of uncertainties in DST predictions. Some key sources of uncertainty in sensor performance and signature propagation predictions have been described, and a probabilistic framework for assessing uncertainty in these predictions was formulated. Application of the framework is challenging, but provides a rational way forward for development of subsequent, practical implementations. A software architecture was sketched by which uncertainty in weather, terrain, and tactical information can be incorporated into DST predictions. Uncertainty is injected at two stages: first, in the creation of an “ensemble” of environmental/tactical scenarios, and second by introducing variations into signature propagation model inputs.

We used two illustrative examples to show the importance of uncertainty in determining appropriate sensor selections, and in identifying the robust characteristics of sensor performance predictions when multiple sources of uncertainty are present. These examples involved a partial implementation of the concepts described in this report. Future work will attempt to implement the full probabilistic framework and software architecture.

Appendix: Probability of Detection Calculations

In this appendix, probabilities of detection and false alarm are calculated for signal and noise with Gaussian pdfs. The derivation allows for possible correlation between the signal and noise. First, let us consider the probability of false alarm, which is given by the integral (e.g., Burdic 1984)

$$P_{fa} = \int_{\gamma}^{\infty} p(n)\,dn. \tag{9}$$

Here, $p(n)$ is the pdf for the noise alone and γ is the user-specified detection threshold. For Gaussian noise, the pdf is

$$p(n) = \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left(-\frac{(n-\mu_n)^2}{2\sigma_n^2}\right), \tag{10}$$

where μ_n and σ_n are the mean and standard deviation, respectively. Substituting Equation 10 into Equation 9, we have

$$P_{fa} = \frac{1}{\sqrt{2\pi\sigma_n^2}} \int_{\gamma}^{\infty} \exp\left(-\frac{(n-\mu_n)^2}{2\sigma_n^2}\right) dn = \frac{1}{2}\operatorname{erfc}\left[(\gamma-\mu_n)/\left(\sqrt{2}\sigma_n\right)\right], \tag{11}$$

in which erfc is the complementary error function.

Next, we consider the probability of detection calculation, which turns out to be considerably more complicated. It requires solution of the following integral:

$$P_d = \int_{\gamma}^{\infty} p(x)\,dx, \tag{12}$$

where:

$x = s + n$ is the sum of the signal and noise
$p(x)$ is its pdf.

Assuming the signal and noise are correlated Gaussian variables, the joint pdf has the form

$$p(s,n) = \frac{1}{2\pi\sigma_s\sigma_n\sqrt{1-\rho_{s,n}^2}} e^{\frac{-1}{2\left(1-\rho_{s,n}^2\right)}\left[\left(\frac{s-\mu_s}{\sigma_s}\right)^2 - 2\rho_{s,n}\left(\frac{s-\mu_s}{\sigma_s}\right)\left(\frac{n-\mu_n}{\sigma_n}\right) + \left(\frac{n-\mu_n}{\sigma_n}\right)^2\right]}, \qquad (13)$$

where:

μ_s and σ_s are the mean and standard deviation, respectively, for the signal alone

$\rho_{s,n}$ is the correlation coefficient between the signal and noise.

To proceed, we must determine $p(x)$ from the joint pdf $p(s,n)$. The pdf for such a sum of two dependent random variables is (Bendat and Pearsol 1986)

$$p(x) = \int_{-\infty}^{\infty} p(s, x-s)ds. \qquad (14)$$

To simplify the integration, we define the following new constants and variables:

$$A = \frac{1}{2\pi\sigma_s\sigma_n\sqrt{1-\rho_{s,n}^2}}, \qquad (15)$$

$$B = \frac{1}{2\left(1-\rho_{s,n}^2\right)\sigma_s^2\sigma_n^2}, \qquad (16)$$

$$s' = s - \mu_s, \qquad (17)$$

$$x' = x - \mu_x, \qquad (18)$$

and

$$\mu_x = \mu_s + \mu_n. \qquad (19)$$

With these definitions, Equation 14 becomes

$$p(x') = Ae^{-BC}\int_{-\infty}^{\infty} e^{-BDs'^2+BFs'}ds', \tag{20}$$

where:

$$C = \sigma_s^2 x'^2 , \tag{21}$$

$$D = \sigma_n^2 + 2\rho_{s,n}\sigma_s\sigma_n + \sigma_s^2 , \tag{22}$$

and

$$F = 2\rho_{s,n}\sigma_s\sigma_n x' + 2\sigma_s^2 x'. \tag{23}$$

According to Gradshteyn and Ryzhik (1980),

$$\int_{-\infty}^{\infty} e^{-u^2x^2 \pm qx}dx = \frac{\sqrt{\pi}}{p}e^{\frac{q^2}{4u^2}} \qquad [\, p > 0\,]. \tag{24}$$

To see if the condition $p > 0$ is met, we consider the coefficient $\sqrt{BD}$ of the x^2 term in the above equation. It is clear that $B > 0$ for $\rho_{s,n}^2 < 1$, and thus there will always be a positive root such that $\sqrt{B} > 0$. Here we must limit ourselves to cases where the signal and the noise are not exactly correlated (either positively or negatively so that the denominator of B does not go to zero); that is, $-1 < \rho_{s,n} < 1$. It can be shown that D is positive for all positive values of σ_n and σ_s and for $|\rho_{s,n}| < 1$, resulting in a real positive root that satisfies the condition above. Using the form specified in Equation 24, the solution to the integral is

$$p(x') = Ae^{-BC}\int_{-\infty}^{\infty} e^{-BDs'^2+BFs'}ds' = A\sqrt{\frac{\pi}{BD}}e^{\frac{BF^2}{4D}-BC}. \tag{25}$$

With some simplification, we find the following Gaussian pdf for *x*:

$$p(x) = \frac{1}{\sigma_x \sqrt{2\pi}} \exp\left(-\frac{\left(x-\mu_x\right)^2}{2\sigma_x^2} \right), \qquad (26)$$

in which the mean is $\mu_x = \mu_s + \mu_n$ and the variance is $\sigma_x^2 = \sigma_n^2 + 2\rho_{s,n}\sigma_s\sigma_n + \sigma_s^2$. Now, the calculation of the probability of detection proceeds exactly as it did for the probability of false alarm, and we have

$$P_d = \frac{1}{\sqrt{2\pi\sigma_x^2}} \int_\gamma^\infty \exp\left(-\frac{\left(x-\mu_x\right)^2}{2\sigma_x^2} \right) dx = \frac{1}{2} \operatorname{erfc}\left[\left(\gamma - \mu_x\right) / \left(\sqrt{2}\sigma_x\right) \right]. \qquad (27)$$

To determine the ROC curve, the threshold γ is varied in Equations 11 and 27 and then P_d is plotted versus P_{fa} for each value of γ.

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14. ABSTRACT

As advanced sensors are increasingly relied upon for force protection, rapid strike, maneuver support, and other tasks, expert decision support tools (DSTs) are needed to recommend appropriate mixes of sensors and placements that will maximize their effectiveness. These tools should predict effects on sensor performance of the many complexities of the environment, such as terrain conditions, the atmospheric state, and background noise and clutter. However, the information available for such inputs is often incomplete and imprecise. To avoid drawing unwarranted conclusions from DSTs, the calculations should reflect a realistic degree of uncertainty in the inputs. In this report, a Bayesian probabilistic framework is developed for this purpose. The initial step involves incorporating uncertainty in the weather forecast, terrain state, and tactical situation by constructing an ensemble of scenarios. Next, a likelihood function for the signal propagation model parameters specifies uncertainty at smaller spatial scales, such as that caused by wind gusts, turbulence, clouds, vegetation, and buildings. An object-oriented software implementation of the framework is sketched. Examples illustrate the importance of uncertainty for optimal sensor selection and determining sensor coverage.

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